3. **Electron Lenses** - let's remember just what a lens does — it deviates a ray by an amount that is proportional to the distance of the ray from the optical axis.

\[ \alpha \propto h \]

\[ \alpha = \frac{h}{f}, \text{ where } f \text{ is the focal length.} \]

3.1 **Refraction.**

boundary separating two regions of different potential.

\[ V_1 \]

\[ v_1 \]

\[ h \]

Conservation of energy: 

\[ eV_1 = \frac{1}{2}mv_1^2 \]

\[ eV_2 = \frac{1}{2}mv_2^2 \implies \frac{v_1}{v_2} = \sqrt{\frac{V_1}{V_2}} \]
The potential changes abruptly at the boundary, but there is no change parallel to it, so that the tangential component of velocity is unchanged.

\[ n \mathbf{e} \cdot V_1 \sin \alpha_1 = V_2 \sin \alpha_2 \]

\[ \Rightarrow \frac{\sin \alpha_1}{\sin \alpha_2} = \frac{V_2}{V_1} = \sqrt{\frac{V_2}{V_1}} \]

which is exactly Snell's law, where \( \sqrt{V} \) may be thought of as the refractive index.

3.2 Lens action

If the boundary is spherical of radius \( R \), and letting the incident ray be parallel to the axis, at some distance \( h \), and assuming that \( h \ll R \) so that \( \sin \alpha = \alpha \),

then deviation of ray, \( \alpha_1 - \alpha_2 = \frac{h}{f} \).

\( \alpha \) deviation \( \propto h \), where \( f \), the constant of proportionality is the focal length as reqd.
Consider parallel rays travelling in opposite directions.

\[ V_1 \]

\[ F_1 \]

\[ V_2 \]

\[ F_2 \]

We have:

\[ \frac{\alpha_1}{\alpha_2} = \sqrt{\frac{V_2}{V_1}} \], \quad \frac{\alpha_3}{\alpha_4} = \sqrt{\frac{V_1}{V_2}} \]

\[ \alpha_1 - \alpha_2 = \frac{h_1}{f_2} \], \quad \alpha_4 - \alpha_3 = \frac{h_2}{f_1} \]

\[ \alpha_1 = \frac{h_1}{R} \], \quad \alpha_3 = \frac{h_2}{R} \]

Eliminating the angles, obtain,

\[ \frac{f_1}{f_2} = \sqrt{\frac{V_1}{V_2}} \]

and note the two focal lengths, \( f_1 \) and \( f_2 \)
3.3 Thick lens. A thick lens is characterized by four focal lengths, of thin lens which has only one.

\[ \frac{1}{P} + \frac{1}{Q} = \frac{1}{f} \]

M = \frac{Q}{P} : magnification

= \frac{r_i}{r_o}

Asymptotic trajectories of electrons in the thick lens can be found as follows:

1. An electron entering the lens parallel to optical axis follows a straight line trajectory to principal plane P2 where the trajectory is refracted such that it leaves the
lens through focal point \( F_2 \).

2\/. An electron going through focal point \( F_1 \) follows a straight line trajectory to principal plane \( P_1 \) and is then refracted such that it leaves the lens parallel to optical axis.

3\/. Trajectories, parallel at the entrance side, cross each other at the same point in the focal plane \( F_2 \), (allowing an arbitrary trajectory to be traced).

\( \text{NB} \) an object in one principal plane is imaged onto the second principal plane with unit magnification, but there will be a change in angle.

From the thick lens geometry we can obtain some useful relationships

\[
(p - F_1)(q - F_2) = f_1 f_2
\]

\[
M = \frac{-f_1}{p - F_1} = - \frac{q - F_2}{f_2}
\]

Thus knowing \( f_1, f_2, F_1 \) and \( F_2 \) we can work out our optical system.
3 cylinder lens $A/0 = 1$. $p = 3, Q = 4$.

$f_1 = 0.897, f_2 = 2.691$
$F_1 = 2.241, F_2 = 0.820$
$V_1 : V_2 : V_3 = 1 : 88 : 9$

$m = 1.18$

\[ \alpha_1 \approx \frac{h}{(p - F_1 + f_1)} \]

\[ \alpha_2 \approx \frac{h}{(Q - F_2 + f_2)} \]

\[ \Rightarrow \alpha_2 = 0.282 \alpha_1 = \frac{\sqrt{g}}{\sqrt{1.18}} \alpha_1 \]
3.4 Practical lenses. 4 common geometries (double cylinder, triple cylinder, double aperture, triple aperture).

3.4.1 Double cylinder:

\[ V_1 \quad V_2 \]

\[ \text{gap, typically } 0.1D \]

\[ \text{diam} = D. \]

\[ \text{the lens properties (e.g. } f, f_1, f_2 \text{) scale with } D \text{ - so all dimensions are in units of } D, \]

\[ \text{typically 10mm.} \]

The two electrodes are held at potentials \( V_1 \) and \( V_2 \) (\( V_1 \neq V_2 \)).

\( V_1 \) corresponds to initial energy.

\( V_2 \) " " final energy.

We have focussed electrons

and changed their energy.
The focal lengths \((f_1, f_2, F_1\) and \(F_2\)) of this lens depend on the ratio \(V_2/V_1\).

\[
\exp \quad \frac{V_2}{V_1} = 8 \quad \Rightarrow \quad F_1 = 1.87 \quad F_2 = 1.47
\]

\[
f_1 = 1.00 \quad f_2 = 2.84
\]

If \(p = 3\) then \(\varphi\) given by

\[(P-F_1)(\varphi-F_2) = f_1 f_2\]

\[\Rightarrow \varphi = 3.98\]

and \(M = \frac{f_1}{P-F_1} = 0.88\)

These dimensions are all in units of \(D\), so if \(D = 5\) mm

then \(P = 15\) mm \(\Rightarrow \varphi = 19.92\) mm.

This information is also given as \(P-\varphi\) curves (Hartvig & Read).

Same example as above:

\[
p = 3 \quad \Rightarrow \varphi \approx 4
\]

\[
\frac{V_2}{V_1} = 8 \quad \Rightarrow \quad M \approx 0.9
\]
The difficulty of not being able to change the overall acceleration ratio without changing the image position is overcome in the triple cylinder lens.

### 3.4.2 Triple cylinder lens.

![Diagram of triple cylinder lens]

- **$V_1$**: corresponds to initial energy
- **$V_3$**: "final"
- **$V_2$**: focusing potential.

Can now change $V_3/V_1$, but keep $p$ and $q$ constant by changing $V_3$ — **zoom lens**.

- **$V_3/V_1 > 1$**: accelerating lens
- **$V_3/V_1 < 1$**: decelerating lens
- **$V_3 = V_1$**: Einzel lens (no energy change but still focusing effect).

The dependence of $V_2/V_1$ (focusing potential) on $V_3/V_1$ (acceleration ratio) is conveniently presented on a zoom lens curve.

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\[ \rho = 5D \quad \alpha = 5D \]
\[ \beta/\alpha = 1 \quad \text{3-cylinder lens} \]

Zoom lens curve.

\[ A: \text{ width of central electrode.} \]

An obvious application of such a lens would be to focus electrons from the target region (with various energies) onto the entrance slit of the energy analyser (fixed pass energy at fixed \( \Delta E \)).

\[ \text{electron primary energy} = V_1 \]

\[ V_2 \quad V_3 = \text{pass energy} \]

\[ \text{target region.} \quad \text{pupil to limit angular range.} \]

\[ \Theta \quad 5D \quad \text{D} \quad 5D. \]

\[ \text{V} = \text{pass energy} = 5 \text{eV} \]
\[ \Rightarrow \frac{V_3}{V_1} = 0.5 \quad \text{from graph} \quad \frac{V_2}{V_1} = 2.3 \quad \text{i.e.} \quad V_2 = 23 \text{volts.} \]

\[ \text{primary energy} = 10 \text{eV} \]

\[ \frac{V_3}{V_1} = 0.25 \quad \text{from graph} \quad \frac{V_2}{V_1} = 1.3 \quad \text{i.e.} \quad V_2 = 13 \text{volts} \]

\[ \text{and if we change primary energy from 10 to 20eV,} \]
\[ \text{then} \quad \frac{V_3}{V_1} = \frac{5}{20} = 0.25 \quad \text{from graph} \quad \frac{V_2}{V_1} = 1.3 \quad \text{i.e.} \quad V_2 = 13 \text{volts} \]

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Some points of note:

1) Range of lens \( \sim x_{10} \text{ to } x_{0.1} \)

\[ \uparrow \text{ accelerating} \uparrow \text{ decelerating} \]

Single lenses can give up to about \( x_{30} \)
for higher acceleration ratios, use combination of
two or more lenses.

The strength of the lens \( \sim \left( \frac{1}{P} + \frac{1}{Q} \right) \sim \frac{1}{F} \)

\( \text{cf. } \frac{1}{F} \text{ in optical lenses (dioptre)}. \)

2) There are 2 values of \( V^3/V_1 \) (focus potential)
for a given \( V^3/V_1 \). Usually prefer to use higher
value of \( V^3/V_1 \), since this minimizes expansion
of the beam in the lens + hence aberrations.

3) As you "zoom" the lens, the linear magnification
will change, and hence transmission of lens. Can
keep this variation below about 10%.

For example \( \left\{ \begin{array}{c}
\text{with } V^3/V_1 = 0.5, M = 1.04 \\
\text{above: } \quad \" = 0.25, M = 1.09
\end{array} \right. \)
Fig. 14 - Actual electron paths in a cylindrical einzel lens operated in the accelerating \( V_2/V_1 = 12.1 \) and decelerating \( V_2/V_1 = 0.045 \) mode (from Adams and Read, 1972). In both modes these lenses have the same first-order focusing properties. However, due to spherical aberrations the image positions are strongly shifted. The length of the central cylinder is equal to the diameter \( D \); the vertical scale is a factor 2.5 larger than the horizontal one. The object distance and paraxial image distance are equal to \( 2D \) (dots). The rays closest to the axis are those of the accelerating mode. The filling factor is 60\% in both cases.
3.4.3 Aperture Lenses

Double-aperture

Apart from their geometry, these lenses have similar properties and uses as cylinder lenses. Cylinder lenses tend to be stronger and have somewhat lower aberrations. On the other hand, aperture lenses are more compact physically.

3.4.4 Four-cylinder Lenses

Recently, four-cylinder lenses have been investigated. Here, $V_1$: initial energy, $V_4$: final energy, but now 2 focusing electrodes. The extra degree of freedom allows you to keep $P$ and $c_f$ constant, as before, but also to keep the linear magnification constant.
TYPICAL CONSTRUCTION OF AN APERTURE LENS

- THREADED ROD
- MOLYBDENUM LENS ELEMENT
- CERAMIC SPACER
- SECURING PLATE
- WASHER
- NUT
- CERAMIC ALIGNMENT ROD
Lens aberrations - due to deviations from the assumption that the rays are paraxial.

Paraxial rays come to a focus at the image plane. Non-paraxial rays come to a focus before the image plane.

The aberration is defined in terms of $\Delta r$.

and $\Delta r = k \Theta^2$ (K given in Hartwig + Read).

Therefore keep $\Theta$ small (i.e. low filling factor of lens)

Filling factor $= 2 \Theta P$/diam. of lens.

Notice that the object achieves a minimum diameter at a point slightly in front of image plane (called circle of least confusion). Therefore in practice can get better throughout by weakening lens slightly (i.e. reduce $V^2/N_1$ by ~10%).
3.6 Matrix methods for ray tracing.

The trajectory of an electron can be characterised by its radial position and its slope, i.e. by the vector $\begin{pmatrix} r \\ \theta \end{pmatrix}$.

- then the various operations on the electron are represented by matrices operating on this vector.

For example, a linear transformation $z_1 \rightarrow z_2$, (an electron drifting in a field free region).

The matrix is $\begin{pmatrix} 1 & (z_2 - z_1) \\ 0 & 1 \end{pmatrix}$

check: $\begin{pmatrix} r_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & (z_2 - z_1) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} r_1 \\ \theta_1 \end{pmatrix}$

giving $r_2 = r_1 + (z_2 - z_1)\theta_1$ and $\theta_2 = \theta_1$.

For a thick lens, imaging between two planes, the matrix is

$$-\frac{1}{f_2} \begin{pmatrix} \Phi - F_2 & (\rho - F_1)(\Phi - F_2) - f_1f_2 \\ 1 & \rho - F_1 \end{pmatrix}$$

- useful procedure when you have combination of several lenses.
SIMION

Simulation of a three-aperture lens.

$A/D = 0.5$

$p = 3D$

$q = 2D$