4. Electrostatic analysers

So far, we have considered collimation and focusing (refraction) of the electron beam. Now consider energy analysis (dispersion)

4.1. Consider a very simple analyser

A parallel plate deflector with electrons injected along the axis

Energy of electrons at entrance slit, \( E_i = eV_i \)

\[ V_i = V_c/2 \]

Electrons are attracted toward positive plate.

\[ F_y = \frac{V_c}{d} \cdot e = \text{const} = m\ddot{y} \Rightarrow \dot{y} = \frac{e}{m} \frac{V_c}{d} \]

Also, \( \frac{1}{2}mv^2 = eV_i \Rightarrow \dot{x} = \left( \frac{2eV_i}{m} \right)^{1/2} \)
time electrons spend between plates = \( L/x = L \left( \frac{m}{2eV_i} \right)^{1/2} \)

\[ \text{displacement at exit} \ y = \frac{1}{2} \left( \frac{(y_i)^2}{t^2} \right) = \frac{1}{2} \left( \frac{e}{m} \frac{V_c}{d} \right) \frac{L^2 m}{2eV_i} \]

NB The ratio \( e/m \) cancels out! So electrostatic analysers work equally well for positive or negative ions of any mass. Of course the heavier the ion, the longer it takes to pass through the system.

Thus \( y = \frac{V_c L^2}{4dV_i} \)

- the displacement is inversely proportional to \( V_i \), the lower the electron energy, the greater the displacement.

The dispersion of the analyser is

\( \frac{dy}{dV_i} = -\frac{V_c L^2}{4dV_i^2} \text{ mm/volt.} \)

- the resolution \( \Delta E \sim \frac{W}{\text{dispersion}} \sim \frac{W}{4dV_i L^2 V_c} \)

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For example, $L = 50\text{mm}$, $d = 10\text{mm}$

$V_c = 4\text{volts}$, $V_i = 5\text{volts}$

$\Rightarrow$ dispersion $= \frac{1 \times 4 \times 2500}{4 \times 25 \times 10} = 10\text{mm/volt}$

and $\Delta E$, for $w = 1\text{mm}$, $= \frac{1}{10} \text{eV}$.

**Limitations of this analyser**

(i) No focusing in $\Theta$ direction, i.e., $y$ depends on $\Theta$

In fact, $y = L \tan \Theta + \frac{L^2}{4d} \frac{V_c}{V_i \cos^2 \Theta}$

So electron (1) has the correct energy, but starts off with a finite angle and misses the slit.

No focusing $\Rightarrow$ loss of signal.

+ electron (2) has wrong energy but because of finite angle passes through slit $\Rightarrow$ degradation of $\Delta E$
(ii) dispersion is non-linear in energy.

(iii) no focussing in $\beta$ direction.

(iv) entrance & exit slit cut through equipotential lines.

4.2 Let's try something else, the same parallel plate deflector but inject electrons at some angle $\theta$ as shown.

\[ y = v \sin \theta, \quad x = v \cos \theta, \quad m \ddot{y} = -\frac{e}{m} \frac{V_c}{d} = \text{const} \]

\[ \therefore \quad \ddot{y} = -\frac{e}{m} \frac{V_c}{d}, \quad \ddot{x} = 0 \]

\[ x = xt, \quad y = \frac{yt - e}{2m} \frac{V_c}{d} t^2 \]
\[ y = 0 \Rightarrow t = \frac{2md}{eV_c}, \quad y' = \frac{v}{x} \]

\[ x = \frac{2md}{eV_c}, \quad y'x = \frac{2md v^2 \sin \Theta \cos \Theta}{eV_c} \]

but \[ \frac{1}{2} m v^2 = eV_i \Rightarrow v^2 = \frac{2V_i e}{m} \]

\[ x = \frac{2md \sin \Theta \cos \Theta}{eV_c} \cdot \frac{2V_i e}{m} = \frac{4V_i d \sin \Theta \cos \Theta}{V_c} \]

\[ x = \frac{2V_i d \sin 2\Theta}{V_c} \]

We require \( x \) to be independent of \( \Theta \), i.e. focussing, i.e. \[ \frac{\partial x}{\partial \Theta} = 0 \]

\[ \frac{\partial x}{\partial \Theta} = 2 \cdot \frac{2V_i}{V_c} d \cos 2\Theta = 0 \Rightarrow \Theta = 45^\circ. \]

Then \( x = \frac{2V_i d}{V_c} \)

\[ \frac{\partial x}{\partial V_i} = \frac{2d}{V_c} \quad \text{dispersion (mm/volt)} \]

\[ \text{linear} \]

So \[ \frac{\partial V_i}{\partial x} = \frac{V_c}{2d}, \quad \text{but } \frac{V_c}{2d} \cdot \frac{L}{L} \Rightarrow \frac{\partial V_i}{\partial x} = \frac{V_i}{L} \]

\[ \therefore \quad \partial V = \frac{V_i}{L} \partial x \]

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\[ \overline{v} = w, \quad V_i = E_i \]
\[ \Delta V = \Delta E_i = \Delta E \]

\[ \therefore \frac{\Delta E}{E_i} = \frac{w}{2L} \quad \text{to first order and} \]

\[ \text{half width} \sim \frac{1}{2} \text{ base width}. \]

\[ \text{independent of } \Theta. \quad \text{re focussing.} \]

Thus \[ \Delta E = E_i \cdot \frac{w}{2L} \quad \text{slit width} \]

\[ \text{pass energy} \quad \text{size of analyser} \]

If you go to 2nd order by using a Taylor expansion you find,

\[ \Delta E = E_i \left( \frac{w}{2L} + \Theta^2 \right) \]

\[ \text{is a term in } \Theta^2, \quad \text{but since } \Theta \text{ is small } \sim 0.1 \text{ rad.} \]

\[ \Theta^2 \ll \Theta \]

So (i) analyser is focussing

(ii) linear dispersion, \[ 8x \propto \Delta E \]

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(iii) electron beam passes through a slit which is parallel to equipotentials

- no focusing in $\beta$ direction.

Solution — bend plates into two concentric cylinders.

In practice the cylinders are complete and the entrance and exit apertures are slits.

NB: $2\pi$ collection in $\beta$ direction $\Rightarrow$ high transmission
4.4 \hspace{1cm} 180^\circ \text{ hemispherical deflector.}

\[ V_2 = V_0 \left( \frac{2R_0 - 1}{R_2} \right) \]

\[ V_1 = V_0 \left( \frac{2R_0 - 1}{R_1} \right) \]

Potential between hemispheres \( \propto \frac{1}{r} \) cf planetary motion.

Electric field \( \propto \frac{1}{r^2} \)

Electrons of energy \( eV_0 \) follow central trajectory.

More energetic electrons less deflected by electric field.

Less energetic electrons more deflected \( \Rightarrow \) dispersion across exit plane in energy.

More than this, the position in the exit plane is independent of \( \Theta \), (just like parallel plate selector).
Also because analyser is symmetric about $\beta$ axis it doesn't matter what angle $\beta$ is – so focussing in two directions — ideally suited to electron beams of cylindrical symmetry.

$$\frac{\Delta E}{E} = \frac{W}{2R_o} + \frac{\Theta^2}{4}$$

$\Delta E$: energy resolution  
$E$: pass energy  
$W$: slit width  
$R_o$: mean radius  
$\Theta$: total angle of beam going into analyser (pencil + beam).

Note the two separate terms.

For optimum combination of transmission and energy resolution make $\Theta^2 = \frac{W}{2R_o}$

So if $R_o = 50\, \text{mm}$, $W = 1\, \text{mm}$ \Rightarrow $\Theta = (\frac{1}{100})^{\frac{1}{2}} = 6^\circ$

then $\frac{\Delta E}{E} = \frac{1}{100} + \frac{1}{400} = 1.3\%$

In the above discussion all defining apertures were considered to be circular. Slits do not alter $\Delta E$ significantly but they do give higher yields — slits must be placed perpendicular to dispersion direction.

Dispersion $\Rightarrow$ dispersion plane.
Computation of electron trajectories in H.A.

\[
\begin{align*}
\sin \theta &= \frac{y}{r} \\
\cos \theta &= \frac{x}{r} \\
r^2 &= x^2 + y^2 \\
F_x &= F(r) \cos \theta \\
F_y &= F(r) \sin \theta \\
E(r) &= -\frac{2V_0R_0}{r^2} \\
F(r) &= -\frac{2eV_0R_0}{r^2}
\end{align*}
\]

\[
\therefore \quad F_x = -\frac{2eV_0R_0 \frac{x}{r^2}}, \quad F_y = -\frac{2eV_0R_0 \frac{y}{r^2}}
\]

\[
\alpha_x = \frac{F_x}{m} = -\frac{2eV_0R_0}{mr^3} \quad \alpha_y = -\frac{2eV_0R_0}{mr^3}
\]

\[
v = v_0 + at, \quad x = x_0 + vt
\]

\[
\therefore \quad v_{x,n+1} = v_{x,n} - \frac{2eV_0R_0}{mr^3} \cdot \alpha_x \Delta t \\
v_{y,n+1} = v_{y,n} - \frac{2eV_0R_0}{mr^3} \cdot \alpha_y \Delta t
\]

\[
x_{n+1} = x_n + \frac{(v_{x,n} + v_{x,n+1}) \Delta t}{2} \\
y_{n+1} = y_n + \frac{(v_{y,n} + v_{y,n+1}) \Delta t}{2}
\]

In the units:

- mm (length)
- ns (time)
- volts (energy)

\[
e/m = 0.1759
\]

\[
v_{x,i} = v_i \sin \alpha \\
v_{y,i} = v_i \cos \alpha
\]

\[
\text{when} \quad v_i = \left(\frac{2eV_i}{m}\right) \frac{1}{2}
\]

and \( eV_i \) is injection energy
4.5 Time resolution of 180° analyser

In the above we saw that the position in the exit plane is independent of θ. However, path 1 is longer than path 2 and electron will take a longer time.

The transit time spread, ΔT, is equal to ~ 20 T₀ when T is time for central trajectory.

Useful relationship \( T(\text{ns}) = \frac{17 d(\text{cm})}{\sqrt{E(\text{eV})}} \)

\[ \text{ex} \]
\[ R = 50 \text{mm}, \quad E = 2 \text{eV} \]
\[ \theta = 3^\circ, \quad T = 200 \text{ns} \quad \text{and} \quad \Delta T \sim 20 \text{ns}. \]

4.6 Field termination in electrostatic analysers.

spherical equipotentials between hemispheres

field badly distorted at entrance + exit planes
(problem alleviated by use of virtual apertures).
Solutions

1) Hoops
2) Jost correctors
3) Wedges.

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**Hoops**

4 concentric hoops, 
potential on each hoop

\[ V = V_0 \left( \frac{2R}{r} - 1 \right) \]

- need additional potentials 
  + feedthroughs.

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**Jost correctors**

3 concentric anulli 
connected to inner & outer 
hemispheres + mean potential. 
Good field termination but 
no additional potentials or 
feedthroughs.

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**Wedges**

The use of additional 
electrodes (e.g. hoops) suggests a continuous 
distribution of electrodes to 
maintain required potential 
distribution (\( \sim \frac{1}{r} \)) \( \Rightarrow \) Wedges. 
particularly important when 
you are looking at the whole of 
the exit plane – multidetection.

Choose shape of wedge to 
approximate \( V(r) \sim \frac{1}{r} \).

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Ceramic substrate with thin carbon coating
4.7 Optimum size of analyser

General system,

\[
\frac{\Delta E}{E_a} = \frac{da}{2R_0} + \frac{x_a^2}{4} \quad - (1)
\]

For optimum transmission/\(\Delta E\) \[
x_a^2 = \frac{da}{2R_0} \quad - (2)
\]

Helmholtz Lagrange \[
x_t E_t^{\frac{1}{2}} d_t = x_a E_a^{\frac{1}{2}} da \quad - (3)
\]

Yield \[
y \propto d_t x_t^2 , \text{ assume } d_t \text{ fixed.}
\]

Then \[
y \propto da^2 \frac{\Delta E}{E_t}
\]

\(\Delta E, E_t\) are fixed by physics, so \(y \propto da^2\)

\(\propto \) proportional to area of entrance slit.
Of course, large values of $d_a$ mean that $R_0$ and $D$ (lens diam) must also be large. Going from $R_0 = 50$ mm to $100$ mm increases $d_a$ by factor of $2$ and yield by $2^2 = 4$.

Of course the magnification of the lens system must match the target diameter to the size of the entrance slit, $d_a$.

Large values of $d_a$ will probably also lead to large values of lens decel. ratios.

NB. These considerations may not be appropriate for angular measurements where $\Delta t$ is fixed.

Limitations to size of analyser:

(i) Size of vacuum vessel
(ii) Magnetic field requirements, $B \leq 5$ mgauss.
(iii) Magnification of lens system.
(iv) Cost.
(v) Weight
(vi) Pumping reqts.
4.8 Tandem analysers

Essentially the same properties as single pass analyser of radius $2R_0$

Advantages:

(i) Twice the dispersion $\rightarrow$ better $\Delta E$ for same pass energy.

(ii) Improved signal/noise

(iii) Improved peak shapes.

(iv) "Straight through" geometry - room for bulky detection systems.

(v) Lighter than equivalent single pass analyser.
Focussing.

Properties of double pass analyser (HDA).

\[ E_1 = 49 \text{ eV} \]
\[ E_2 = 50 \text{ eV} \]
\[ E_3 = 51 \text{ eV} \]

Energy dispersion.
4.9 Non linearities in dispersion

$x$ is the radial distance of the electron from the path of radius $R_0$ and let $\Delta E = E - E_0$.

Then, in general:

$$\frac{x_2}{R_0} = -\frac{x_1}{R_0} + 2\frac{\Delta E}{E_0} + 2\left(\frac{\Delta E}{E_0}\right)^2 - 2 \chi^2$$

First term: shows magnification of HODA = -1

Second term: shows linear dispersion

Third term: shows non-linearity in dispersion

Fourth term: shows angular aberration. (tails)

* especially important for multidetectors
\( \Delta E \) 

\[ \chi^2 \text{ term, moves centre of gravity of peak} \]

linear dispersion

non-linear (exaggerated)
4.10 Example of the design of an electron spectrometer

Input parameters

- Range of K.E
  at target: 1 - 20 eV
- $\Delta E$: $\sim$ 25 meV
- $\Delta \Theta$: $\sim$ 1.5°
- Object diam: 1 mm.

Constraints

- $R_0$: 50 mm
- Dist. from target to ITOA entrance slit: 100 mm
- Working dist. > 25 mm

Entrance slit $w$: Size of target is 1 mm + if magnification of lens is say 1 $\Rightarrow w = 1$ mm.

Is this reasonable?

$$\frac{\Delta E}{E_{pass}} = \frac{w}{2R_0} + \frac{\alpha^2}{4} \text{ with } \alpha^2 \leq \frac{w}{2R_0}$$

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\[ \therefore \frac{x^2}{100} \leq \frac{1}{100} \Rightarrow x \leq \frac{1}{10} \text{ rad} \leq 5^\circ \]

and \[ \frac{\Delta E}{E_{\text{pass}}} = \frac{1}{100} + \frac{1}{400} \approx 1.25\% \]

ie \[ 25\text{ meV} @ E_{\text{pass}} = 2\text{eV} \]
\[ 12.5\text{ meV} @ E_{\text{pass}} = 1\text{eV} \]

\{ sounds reasonable. \}

Max. decoll. reqd \[ \sim \frac{20}{2} \sim \times 10. \] \[ \text{larger KE's would req. larger ratios + hence stronger lens.} \]

This would be provided by the zoom lens described earlier, with \[ P = 50, \ \Phi = 50 \text{ and } \frac{A}{D} = 1.0 \]

and with mag. of 1 as reqd.

\[ P + \Phi = 10 \quad D = 100\text{mm} \Rightarrow D = 10\text{mm} \]

\underline{Angle defining aperture}

This should be placed close to focal plane of lens to make beam angle = zero at entrance plane of H.O.A. Accomplished by putting this aperture at 2.50 from lens centre. This is well away from the
changing field of the lens, and also helps to stop field penetration from the middle element of the lens.

Also, the working distance is \(2.50 = 25 \text{mm}\) as reqd.

\[
\Theta p = \frac{r_p}{25 \text{mm}}
\]

For \(r_p = \frac{1}{2} \text{mm}\), \(\Theta p \approx 1^\circ\)

Remembering that target has finite size (1 mm),

total angle (pencil + beam angle) will be \(\sqrt{5} \approx 1^\circ\)

\(\approx 1.5^\circ\) as reqd.

Angle at entrance to HDA, \(\chi_a\) at analyser entrance.

\[
H.L.: \quad r_t \chi_t \sqrt{E_t} = r_a \chi_a \sqrt{E_{\text{pass}}} \implies \chi_a = \frac{1}{M \sqrt{E_t / E_{\text{pass}}}}
\]

at target

\(\approx 3 \chi_t \approx 3^\circ \leq 5^\circ\)

Filling factor of lens \(\approx 0.5 + \frac{50}{57} \approx 30\%\).
electrostatic shield

target region.

relationship comes from zoom lens curve

ramp (a)

Focus P.S.

Kinetic energy P.S.

pass energy

cem front end

inner

outer

hemispheres

beam dump

gap = 28 mm

50 mm

deflector (power supply read)

EHT+ signal

Jost correctors

ramp (b)

ZERO OF ELECTRON ENERGY
(all voltages to be measured with respect to this rail!)

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Single channel detection.

Multichannel detection.
Points of note:

**Constructional materials:** molybdenum, HCOF copper alumin. with graphite coating

**Power supplies:** hum + ripple < few mV.

**Magnetic field:** ≤ few mgauss.
- mumetal shielding, helmholtz coils.

**Detector:** single channel, channel elec. multiplier, multidection, (large gap) note bias + mesh

**Other values of P + Q**

**Scanning electron kinetic energy.**

**Deflectors.**

**Baking** ~ 100 - 150°C

**Electrostatic shield at target**

**Beam dump.**