A charged particle constrained to move in curved path experiences a centripetal acceleration. Due to this acceleration, the particle radiates energy according to Maxwell equations. A non-relativistic particle emits radiation primarily at its frequency of revolution, with the characteristic pattern shown in figure 1(a). However, as the speed of the particle approaches the speed of light, the radiation pattern is distorted by relativistic effects and changes to a narrow cone of radiation with angular spread $\Delta \phi$. The latter type of radiation is called synchrotron radiation.

![Diagram](image1.png)

**Figure 1.** The radiation pattern for (a) a charged particle traversing a circular trajectory with non-relativistic velocity and (b) the radiation pattern of a charged particle moving at relativistic speed.

The properties of synchrotron radiation are well known and some of them are summarised here. For a particle moving at velocity $v$ with a total energy $E$:

$$E = \gamma m_e c^2 \quad \text{Eq. (1)}$$
with
\[
\gamma = \frac{1}{\sqrt{1 - \frac{v}{c^2}}}
\]
Eq (2)

the opening angle of the radiation cone shown in figure 3.29(b) can be expressed, for high values of \( \gamma \) as:

\[\Delta \phi \equiv \gamma^{-1}\]

Therefore, for high values of \( \gamma \), a well-collimated fan of radiation is produced. For an observer located in the laboratory frame of reference, the radiation emitted by the charged particle consists of a series of short pulses of light. Each pulse is separated from each other by a period equal to the transit time of the electron in the circular path. It can be shown, using Fourier analysis, that the spectrum of synchrotron radiation produced by the charged particle extends from the frequency of revolution, \( \omega \), to frequencies of higher harmonics. In practice, however, the electron orbit frequency is not precisely given, because a charged particle confined to follow a circular orbit oscillates in position and energy about its equilibrium orbit. Thus, the spectrum of even a single particle will be smeared out into a continuum. This implies that synchrotron radiation sources produce photons with a continuum of energies (or colors), from the infrared, to the X-Ray region. Monochromatic radiation of the desired wavelength can be obtained by locating a monochromator at a tangential point of the circular orbit, where the radiation is emitted. The spectral range of photons produced by electrons following a path with a radius of curvature \( R_b \) can be characterised with the critical energy \( E_c \) where:

\[E_c = \frac{3c\gamma^3}{2R_b}\]

\( E_c \) is defined in terms of the spectral power radiated by a relativistic particle: half of the power spectra is radiated at energies below \( E_c \) and the other half at energies above it. Thus, the energy of the electrons circulating the storage ring in a synchrotron radiation facility determines the optimum ranges of energies available.
Useful references: